



## Laser Cooling of TeV Muons\*

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We show that Compton scattering can be used to cool TeV-scale muon beams, and derive analytical expressions for the equilibrium transverse angular spread, longitudinal energy spread, and power requirements. We find that a factor of a few thousand reduction in emittance is possible for a 3 TeV muon collider.

**1. Introduction**

Muon colliders are a possible future tool for the exploration of physics at the TeV scale and beyond. The current status of the development of the muon collider concept is described in Ref. [1], which includes a description of a 3 TeV center-of-mass (COM) machine.

One of the major challenges to realizing a high-luminosity muon collider is to cool the diffuse bunches of muons produced in pion decays. Much effort is being put into the problem of quickly cooling these bunches at the front-end of a muon-collider and maintaining the low emittance while bunches are being accelerated and brought into collision.

For the 3 TeV COM machine, the problems of neutrino radiation [2] and power consumption are already becoming prohibitive. Additional cooling of the bunches after acceleration would mitigate these problems by allowing a given luminosity to be attained with fewer stored muons and a lower repetition rate, and allow consideration of even higher energy muon colliders. For example, the tunnels and high-field magnets being discussed for future hadron colliders [3] could ultimately be used for a 100 TeV-scale muon collider. Post-acceleration cooling would also reduce detector backgrounds from muon decay.

The possibility of using Compton scattering for cooling of electron bunches for  $\gamma\gamma$  colliders has been previously considered [4]. The luminosity for  $e^+e^-$  collisions is already limited by beamstrahlung effects, but additional cooling may greatly improve the  $\gamma\gamma$  luminosity.

We propose herein the possibility of post-acceleration cooling of muons beams using Compton scattering.

**2. Compton Scattering**

The Compton Scattering cross-section, in the rest frame of the muon, is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2m^2} \left( \frac{k'}{k} \right)^2 \left( \frac{k'}{k} + \frac{k}{k'} - \sin^2 \theta \right), \quad (1)$$

where

$$k' = \frac{k}{1 + (k/m)(1 - \cos \theta)} \quad (2)$$

and  $k$  is the incoming photon energy,  $k'$  is the outgoing photon energy, and  $\theta$  is the photon scattering angle. For  $k \ll m$ ,  $k' \approx k$  and the scattering is roughly isotropic:

$$\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta), \quad (3)$$

and the total cross-section is given by:

$$\sigma_C = \frac{8\pi}{3} \frac{\alpha^2}{m^2} = \frac{8\pi}{3} r_\mu^2 \quad (4)$$

where  $r_\mu^2$  is the classical muon radius. Thus, compared to electrons, the cross-section is reduced by  $\approx 4 \times 10^4$ .

As an example, to expect 1 collision with a 0.1 eV photon, the light energy density would need to be  $10 \text{ J}/(\mu\text{m})^2$ .

Consider a beam of muons, with energy and momentum defined by  $\beta$ ,  $\gamma$ ,  $E_\mu$ , and  $p_\mu$ , colliding head-on with a mono-energetic beam of photons

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with energy  $E_\gamma$ . We will approximate  $\beta = 1$ . The energy of the photons in the muon rest frame is:

$$E_\gamma^* = \gamma E_\gamma (1 + \beta) \approx 2\gamma E_\gamma. \quad (5)$$

On average, the photon will transfer longitudinal momentum  $E_\gamma^*$  to the muon. In the lab frame,

$$E_\mu \rightarrow E_\mu - 2\gamma^2 E_\gamma. \quad (6)$$

Typically, the muon will receive a smaller transverse kick:

$$p_{T\mu} \approx \gamma E_\gamma. \quad (7)$$

Therefore, for large  $\gamma$ , the muon is essentially slowed down without changing direction.

If lower photon energies are needed, this can effectively be achieved by aiming the photon beam at an angle  $\theta$  relative to the head-on direction. The transverse momentum of the photon is inconsequential. The effective photon energy becomes:

$$E_\gamma \rightarrow E_\gamma (1 + \cos \theta)/2. \quad (8)$$

### 3. Cooling Effect

For simplicity, we consider the case that the muons undergo on average one Compton scattering, and are afterwards reaccelerated to compensate for the average energy loss. The following conclusions are also valid for any average number of scatterings, as long as the relative energy loss is small.

Transversely, a muon has a small angle  $\alpha$  relative to the beam direction in a plane, for example the  $x - z$  plane. After the scattering,  $\alpha$  remains unchanged, within an amount  $E_\gamma/m$ . After reacceleration,

$$\alpha \rightarrow \alpha (1 - 2\gamma E_\gamma/m). \quad (9)$$

Thus, the angular spread can be reduced, in the same way as for ionization cooling at low energy.

There is also a heating effect from the spread in the transverse kick. The average increase in the variance of  $\alpha$  is equal to that for one Compton scattering:

$$\sigma_\alpha^2 \rightarrow \sigma_\alpha^2 + \frac{6}{5} \left( \frac{E_\gamma}{m} \right)^2. \quad (10)$$

To find the equilibrium angular spread we equate the cooling and heating effects to find:

$$\sigma_\alpha = \sqrt{\frac{3}{10} \frac{E_\gamma}{E_\mu}}. \quad (11)$$

Luminosity is inversely proportional to the sums of the emittances of the two colliding beams. It is also proportional to the product of the numbers of muons in the two beams. This product decays with a time constant of one-half the muon lifetime. For a fractionally small energy loss per collision, the number of Compton scatterings needed to reduce  $\sigma_\alpha$  by a factor of  $1/e$  is given by:

$$n = \frac{m}{2\gamma E_\gamma} \quad (12)$$

The total energy used to reaccelerate the muon after these  $n$  collisions is equal the original muon energy:

$$E_{reacc} = E_\mu. \quad (13)$$

The power density needed to attain this factor in one-half muon lifetime is given by:

$$p = \frac{n E_\gamma}{\sigma_C \gamma \tau_\mu / 2} = \frac{3}{8\pi} \frac{m}{\gamma^2 \tau_\mu r_\mu^2}. \quad (14)$$

Thus, the power required decreases as the square of the muon energy increases.

Longitudinally, since the average energy loss in a Compton scattering is greater the higher the muon energy, there is an energy bunching effect. For  $\sigma_{E_\mu} \ll E_\mu$ , and the average case of one Compton scattering, the bunching effect is:

$$\sigma_{E_\mu} \rightarrow \sigma_{E_\mu} - \frac{\sigma_{E_\mu}}{E_\mu} 4\gamma^2 E_\gamma. \quad (15)$$

There are also two sources of energy heating. The first is from the variance in the number of Compton scatterings of the muon, given by Poisson statistics. This leads to a variance in the energy spread of  $(2\gamma^2 E_\gamma)^2$ . The second is from the variance in the energy spread within one Compton scattering, given by  $2/5 (2\gamma^2 E_\gamma)^2$ . The total heating effect is then:

$$\sigma_{E_\mu}^2 \rightarrow \sigma_{E_\mu}^2 + (2\gamma^2 E_\gamma)^2 + \frac{2}{5} (2\gamma^2 E_\gamma)^2. \quad (16)$$

Equating the heating and cooling effects, we find for the equilibrium energy spread:

$$\frac{\sigma_{E_\mu}}{E_\mu} = \sqrt{\frac{7}{10} \frac{E_\mu E_\gamma}{m^2}}. \quad (17)$$

We have checked these derivations with a simple Monte Carlo simulation of a set of muons undergoing repeated Compton scatterings and boosts. The predictions of these equations are in excellent quantitative agreement with the simulation.

#### 4. Power Considerations

In principle, Compton scattering could be used to cool low-energy muons. Unfortunately, our estimates show that the power requirements would be prohibitive by many orders of magnitude. However, as shown in Eq. 14, the power density needed decreases as the square of the muon energy increases, and may be reasonable at TeV energies.

Two other general considerations affect how the power needed scales with the muon energy. First, the muons will be in a storage ring. The photon pulses can be placed in a cavity and reused once per turn of the muons around the ring, as illustrated in Fig. 1. The size of the ring, and therefore the time per turn, is proportional to the muon energy. For a lower muon energy, the photon pulses can be reused at a faster rate. Therefore, for this scheme, the power needed to produce the photons scales as only  $1/E$ .

The second consideration is that the geometric emittance, and therefore spot size, decreases linearly as the muon bunch is accelerated. Therefore, the area that needs to be illuminated, and the total power, decreases as the square of the muon energy. Putting these two considerations together, the power needed for cooling scales as  $1/E^3$ .

The length of the muon bunch can also affect the amount of power needed. A laser beam can be focussed to collide with the muon bunch. The diffraction-limited spot size at the focus is proportional to the F-stop ( $F$ ). However, the depth of focus is proportional to  $F^2$ . If the length of the muon bunch is too long, we will need to increase

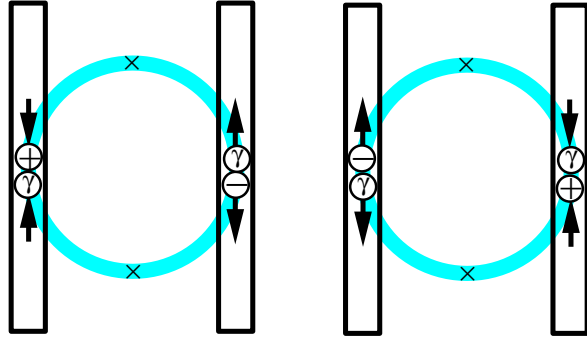


Figure 1. Configuration of resonant cavities and muon storage ring. On the left, each muon bunch collides head-on with a photon pulse. On the right, one-half cycle later, each photon pulse has reflected and collides head-on with the other muon bunch.

the depth of focus, which implies increasing the spot size, which will require proportionally more total power.

Finally, Eq. 14 gives only the cooling rate needed to compensate the luminosity for the muon decays. A cooling rate several times higher than this will be necessary to realize large increases in the luminosity. We also note that as cooling power is added, we also need to add RF power to reaccelerate the muons, as described in Eq. 13.

Although a lot of additional power will be needed for the laser and RF systems, a much lower repetition rate will be needed for a given luminosity, and thus the power consumption for the facility as a whole may be much lower.

#### 5. Application to the 3 TeV Muon Collider

We start with the parameters for the 3 TeV muon collider in Ref. [1]. The ring has a circumference of 6 km, and contains 4 bunches at a time. We assume that this will be reduced to 1 bunch. We have assumed that the bunch length can be shortened to 1 mm. Midway between each crossing point, we place a resonant optical cavity. This is illustrated in Fig. 1. Each cavity contains a photon pulse reflecting back and forth. The

Table 1

Parameters of the 3 TeV COM muon collider and possible laser cooling systems. These parameters assume that the full emittance improvement will be attained in one luminosity lifetime.

|  | Ref. [1]           | CO <sub>2</sub> laser | Nd:Glass Laser     |
|--|--------------------|-----------------------|--------------------|
| Bunches/fill                                 | 4                  | 1                     | 1                  |
| Rep. rate (Hz)                               | 15                 | 1                     | 1                  |
| Initial beam width ( $\mu\text{m}$ )         | 3.2                | 3.2                   | 3.2                |
| Initial bunch length (mm)                    | 3.0                | 1.0                   | 1.0                |
| $\lambda$ ( $\mu\text{m}$ )                  |                    | 10.6                  | 1.05               |
| $E_\gamma$ (eV)                              |                    | 0.1                   | 1.0                |
| Emittance improvement                        |                    | 7000                  | 2200               |
| Energy spread (%)                            | 0.16               | 0.3                   | 1.0                |
| F-stop                                       |                    | 7                     | 22                 |
| Diffraction-limited width ( $\mu\text{m}$ )  |                    | 43                    | 13                 |
| Energy in photons (MJ)                       |                    | 4                     | 0.4                |
| Efficiency (%)                               |                    | 25                    | 1                  |
| $\mu/\text{bunch}$                           | $2 \times 10^{12}$ | $3 \times 10^{11}$    | $5 \times 10^{11}$ |
| Tune shift                                   | 0.044              | 31                    | 15                 |
| Luminosity ( $\text{cm}^{-2}\text{s}^{-1}$ ) | $7 \times 10^{34}$ | $7 \times 10^{34}$    | $7 \times 10^{34}$ |

length of the cavity is set to 3 km, so that the light pulse hits a muon bunch in alternating directions once per reflection. The luminosity lifetime is 15 msec, or 750 turns. Thus, the cavity should have a  $Q$  of  $\approx 10^3$ . The length of the photon pulse should be comparable to the  $\beta^*$  of the machine, or about 30 ps.

Progress in improving laser intensity and decreasing photon pulse widths has been very rapid [5]. For example, the Mercury project [6] is developing a  $1.05 \mu\text{m}$  laser system that will generate 100 J at 10 Hz in 5 ns pulses at 10% efficiency. We have considered two types of laser systems to generate the necessary photon pulses: CO<sub>2</sub> lasers, which typically have a  $10.6 \mu\text{m}$  wavelength and 25% efficiency, and Nd:Glass lasers, which typically have a  $1.05 \mu\text{m}$  wavelength and 1% efficiency. The parameters of these laser systems with the 3 TeV COM machine are shown in Table 1.

As shown in Table 1, we can expect emittance improvements by a factor of a few thousand. However, the energy needed in the laser

pulses is very high: Of order one MJ. The two laser cooling stations in Fig. 1 would probably need to be divided into several cooling stations each with a fraction of the laser energy. Shortening the muon bunch length could also allow a considerable reduction in laser energy. It may be possible to exploit the reduction in beam size that occurs during the cooling process. Finally, a higher energy muon collider would also reduce the laser energy requirement.

Some of the other parameters of the collider with laser cooling may present challenges for the machine design. The small emittance leads to a very high value of the tune shift. Also, the energy spreads are somewhat high, especially for the lower-wavelength laser.

## 6. Conclusions

We have shown that Compton scattering can be used to cool muon beams. Eq. 11 and Eq. 17 describe the achievable transverse angular spread and longitudinal energy spread.

While the power needed for laser cooling of

muon beams is very high, it decreases as the cube of the muon energy increases, and may become practical by TeV energies. If so, emittance reductions by factors of a few thousand are possible. This would allow a given luminosity to be attained with a much lower repetition rate, much less detector background from muon decays, and a much reduced neutrino radiation hazard. Several challenges remain to develop a plausible optical, laser, and machine system.

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